MAA

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

66th Annual

AMC 12 A

American Mathematics Contest 12 A

Tuesday, February 3, 2015

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 33rd annual American Invitational Mathematics Examination (AIME) on Thursday, March 19, 2015 or Wednesday, March 25, 2015. More details about the AIME and other information are on the back page of this test booklet.

1. What is the value of $(2^0 - 1 + 5^2 + 0)^{-1} \times 5$?

   (A) -125 **(B)** -120 **(C)** $\frac{1}{5}$ **(D)** $\frac{5}{24}$ **(E)** 25

2. Two of the three sides of a triangle are 20 and 15. Which of the following numbers is not a possible perimeter of the triangle?

   (A) 52 **(B)** 57 **(C)** 62 **(D)** 67 **(E)** 72

3. Mr. Patrick teaches math to 15 students. He was grading tests and found that when he graded everyone's test except Payton's, the average grade for the class was 80. After he graded Payton's test, the class average became 81. What was Payton's score on the test?

   (A) 81 **(B)** 85 **(C)** 91 **(D)** 94 **(E)** 95

4. The sum of two positive numbers is 5 times their difference. What is the ratio of the larger number to the smaller?

   (A) $\frac{5}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{5}$ **(D)** 2 **(E)** $\frac{5}{2}$

5. Amelia needs to estimate the quantity $\frac{a}{b} - c$, where a, b, and c are large positive integers. She rounds each of the integers so that the calculation will be easier to do mentally. In which of these situations will her answer necessarily be greater than the exact value of $\frac{a}{b} - c$?

   (A) She rounds all three numbers up.

   (B) She rounds a and b up, and she rounds c down.

   (C) She rounds a and c up, and she rounds b down.

   (D) She rounds a up, and she rounds b and c down.

   (E) She rounds c up, and she rounds a and b down.

6. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

   (A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

7. Two right circular cylinders have the same volume. The radius of the second cylinder is 10% more than the radius of the first. What is the relationship between the heights of the two cylinders?

   (A) The second height is 10% less than the first.

   (B) The first height is 10% more than the second.

   (C) The second height is 21% less than the first.

   (D) The first height is 21% more than the second.

   (E) The second height is 80% of the first.

8. The ratio of the length to the width of a rectangle is $4:3$. If the rectangle has diagonal of length d, then the area may be expressed as kd^2 for some constant k. What is k?

   (A) $\frac{2}{7}$ **(B)** $\frac{3}{7}$ **(C)** $\frac{12}{25}$ **(D)** $\frac{16}{25}$ **(E)** $\frac{3}{4}$

9. A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?

   (A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

10. Integers x and y with $x > y > 0$ satisfy $x + y + xy = 80$. What is x?

    (A) 8 **(B)** 10 **(C)** 15 **(D)** 18 **(E)** 26

11. On a sheet of paper, Isabella draws a circle of radius 2, a circle of radius 3, and all possible lines simultaneously tangent to both circles. Isabella notices that she has drawn exactly $k \geq 0$ lines. How many different values of k are possible?

    (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. The parabolas $y = ax^2 - 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area 12. What is $a + b$?

    (A) 1 **(B)** 1.5 **(C)** 2 **(D)** 2.5 **(E)** 3

13. A league with 12 teams holds a round-robin tournament, with each team playing every other team exactly once. Games either end with one team victorious or else end in a draw. A team scores 2 points for every game it wins and 1 point for every game it draws. Which of the following is *not* a true statement about the list of 12 scores?

(A) There must be an even number of odd scores.

(B) There must be an even number of even scores.

(C) There cannot be two scores of 0.

(D) The sum of the scores must be at least 100.

(E) The highest score must be at least 12.

14. What is the value of a for which $\dfrac{1}{\log_2 a} + \dfrac{1}{\log_3 a} + \dfrac{1}{\log_4 a} = 1$?

(A) 9 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 36

15. What is the minimum number of digits to the right of the decimal point needed to express the fraction $\dfrac{123\,456\,789}{2^{26} \cdot 5^4}$ as a decimal?

(A) 4 **(B)** 22 **(C)** 26 **(D)** 30 **(E)** 104

16. Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\dfrac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\dfrac{24}{5}\sqrt{2}$

17. Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\dfrac{47}{256}$ **(B)** $\dfrac{3}{16}$ **(C)** $\dfrac{49}{256}$ **(D)** $\dfrac{25}{128}$ **(E)** $\dfrac{51}{256}$

18. The zeros of the function $f(x) = x^2 - ax + 2a$ are integers. What is the sum of the possible values of a?

(A) 7 **(B)** 8 **(C)** 16 **(D)** 17 **(E)** 18

19. For some positive integers p, there is a quadrilateral $ABCD$ with positive integer side lengths, perimeter p, right angles at B and C, $AB = 2$, and $CD = AD$. How many different values of $p < 2015$ are possible?

(A) 30 **(B)** 31 **(C)** 61 **(D)** 62 **(E)** 63

20. Isosceles triangles T and T' are not congruent but have the same area and the same perimeter. The sides of T have lengths of 5, 5, and 8, while those of T' have lengths a, a, and b. Which of the following numbers is closest to b?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

21. A circle of radius r passes through both foci of, and exactly four points on, the ellipse with equation $x^2 + 16y^2 = 16$. The set of all possible values of r is an interval $[a, b)$. What is $a + b$?

(A) $5\sqrt{2}+4$ **(B)** $\sqrt{17}+7$ **(C)** $6\sqrt{2}+3$ **(D)** $\sqrt{15}+8$ **(E)** 12

22. For each positive integer n, let $S(n)$ be the number of sequences of length n consisting solely of the letters A and B, with no more than three As in a row and no more than three Bs in a row. What is the remainder when $S(2015)$ is divided by 12?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

23. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers and $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

24. Rational numbers a and b are chosen at random among all rational numbers in the interval $[0, 2)$ that can be written as fractions $\frac{n}{d}$ where n and d are integers with $1 \le d \le 5$. What is the probability that

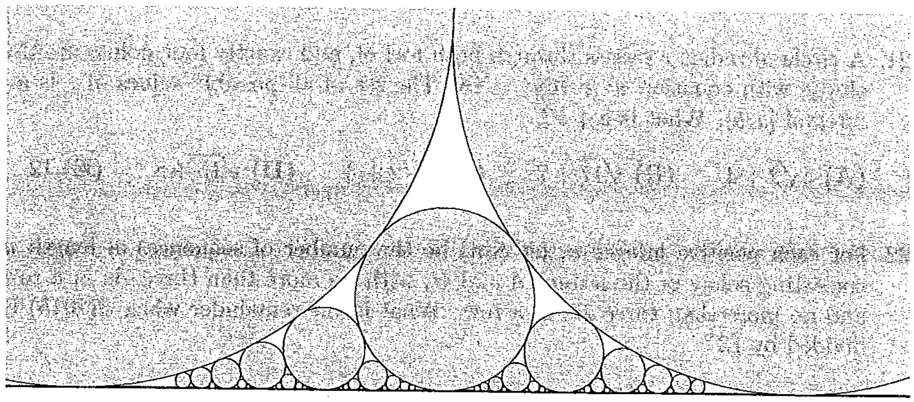
$$(\cos(a\pi) + i\sin(b\pi))^4$$

is a real number?

(A) $\frac{3}{50}$ **(B)** $\frac{4}{25}$ **(C)** $\frac{41}{200}$ **(D)** $\frac{6}{25}$ **(E)** $\frac{13}{50}$

25. A collection of circles in the upper half-plane, all tangent to the x-axis, is constructed in layers as follows. Layer L_0 consists of two circles of radii 70^2 and 73^2 that are externally tangent. For $k \geq 1$, the circles in $\bigcup_{j=0}^{k-1} L_j$ are ordered according to their points of tangency with the x-axis. For every pair of consecutive circles in this order, a new circle is constructed externally tangent to each of the two circles in the pair. Layer L_k consists of the 2^{k-1} circles constructed in this way. Let $S = \bigcup_{j=0}^{6} L_j$, and for every circle C denote by $r(C)$ its radius. What is

$$\sum_{C \in S} \frac{1}{\sqrt{r(C)}}?$$



(A) $\frac{286}{35}$ (B) $\frac{583}{70}$ (C) $\frac{715}{73}$ (D) $\frac{143}{14}$ (E) $\frac{1573}{146}$

MATHEMATICAL ASSOCIATION OF AMERICA

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WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 and orders for publications should be addressed to:

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The problems and solutions for this AMC 12 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 12 Subcommittee Chair:

Jerrold W. Grossman

2015 AIME

The 33rd annual AIME will be held on Thursday, March 19, with the alternate on Wednesday, March 25. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 44th Annual USA Mathematical Olympiad (USAMO) on April 28–29, 2015. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: maa.org/math-competitions

2015

AMC 12 A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 3, 2015

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 3, 2015. Nothing is needed from inside this package until February 3.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 12 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

The

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